An Implicit Monte Carlo Acceleration Scheme

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Abstract and Prior Work

IMC [1] runtimes are dominated by photons in thick media where effective scattering is high. Our hybrid method removes effective scattering which makes the cost independent of MFP. Prior work includes:

Stochastic	Deterministic	Hybrid
F&C RW [2]	VEF [5]	C&L WW [8]
IMD [3]	DSA [6]	MC HOLO [9]
DDMC [4]	SMM [7]	

The Hybrid Moment Method

Following Olivier [10], the model problem is,

$$\boldsymbol{\Omega}\cdot\nabla\psi + \sigma_t\psi = \frac{\sigma_s}{4\pi}\int \psi \,\mathrm{d}\Omega' + q\,, \quad \mathrm{x}\in\mathcal{D}\,, \tag{1a}$$

$$\psi(\mathrm{x},\boldsymbol{\Omega}) = \bar{\psi}(\mathrm{x},\boldsymbol{\Omega}), \quad \mathrm{x}\in\partial\mathcal{D} \text{ and } \boldsymbol{\Omega}\cdot\mathrm{n} < 0\,. \tag{1b}$$

The moment system is,

$$\nabla\cdot\boldsymbol{J} + \sigma_a\varphi = Q_0\,, \quad \mathrm{x}\in\mathcal{D}\,, \tag{2a}$$

$$\frac{1}{3}\nabla\varphi + \sigma_t\boldsymbol{J} = \boldsymbol{Q}_1 - \nabla\cdot\mathrm{T}\,, \quad \mathrm{x}\in\mathcal{D}\,, \tag{2b}$$

$$\boldsymbol{J}\cdot\mathrm{n} = \frac{1}{2}\varphi + 2J_{\mathrm{in}} + \beta\,, \quad \mathrm{x}\in\partial\mathcal{D}\,. \tag{2c}$$

The correction tensor and boundary factor are,

$$\mathrm{T} = \int \boldsymbol{\Omega}\otimes\boldsymbol{\Omega}\psi\,\mathrm{d}\Omega - \frac{1}{3}\mathrm{I}\int\psi\,\mathrm{d}\Omega\,, \tag{2d}$$

$$\beta = \int |\boldsymbol{\Omega}\cdot\mathrm{n}|\psi\,\mathrm{d}\Omega - \frac{1}{2}\int\psi\,\mathrm{d}\Omega. \tag{2e}$$

We solve the 2nd order form, found by eliminating J,

$$-\nabla\cdot\frac{1}{3\sigma_t}\nabla\varphi + \sigma_a\varphi = Q_0 - \nabla\cdot\frac{\boldsymbol{Q}_1}{\sigma_t} + \boxed{\nabla\cdot\frac{1}{\sigma_t}\nabla\cdot\mathrm{T}}\,, \tag{3}$$

where the boxed term is addressed in Future Work. Figure 1 shows the iteration.

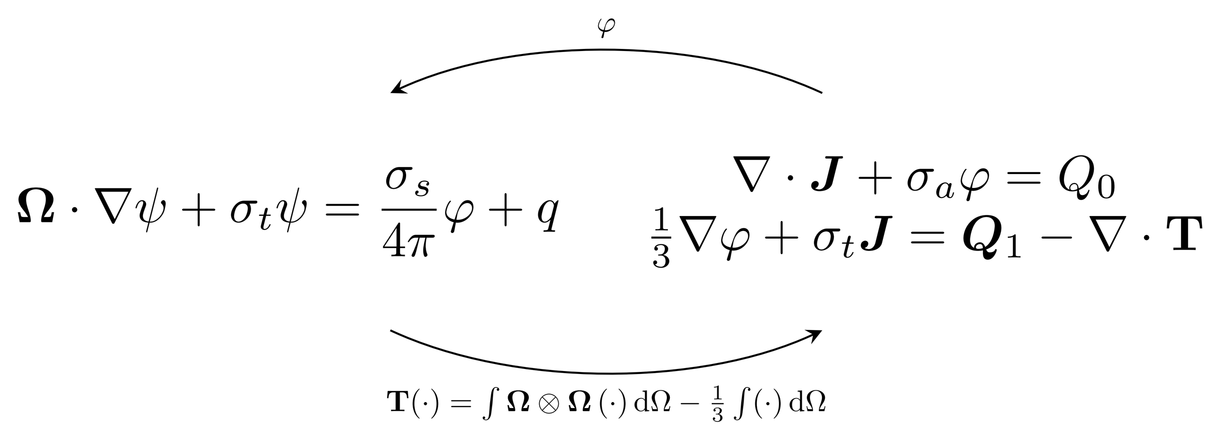


Figure 1: SMM iteration, reproduced from Olivier [10].

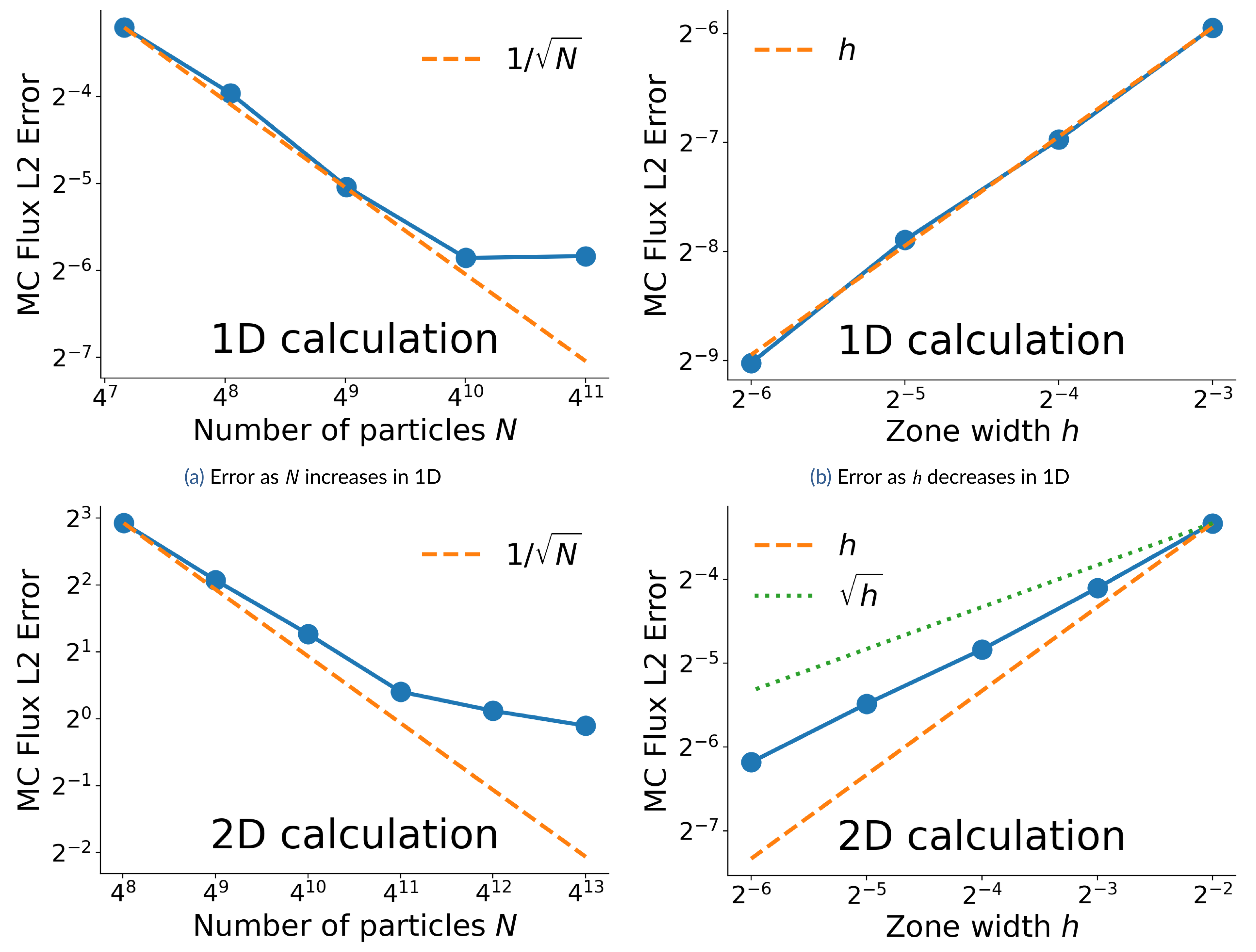
Compute $\hat{\mathrm{T}} = \hat{\mathrm{P}} - \frac{1}{3}\mathrm{I}\hat{\phi}$ and $\hat{\beta} = \hat{K} - \frac{1}{2}\hat{\phi}$ using 3 tallies,

$$\hat{\phi} = \frac{\sum_i d_i w_i}{V},\quad \hat{\mathrm{P}} = \frac{\sum_i \boldsymbol{\Omega}_i\otimes\boldsymbol{\Omega}_i d_i w_i}{V},\quad \hat{K} = \frac{\sum_i |\boldsymbol{\Omega}_i\cdot\mathrm{n}| d_i w_i}{V}. \tag{4}$$

Result #1: Error goes like $C_0/\sqrt{N} + C_1 h$ in 1D but not in 2D problem

Figure 2 shows that the error in 1D fits our prediction but not in 2D. The error is with respect to the MMS solutions,

$$\psi_{\text{1D}} = (1+\mu+\mu^2)\sin\pi x, \quad \psi_{\text{2D}} = \alpha + \boldsymbol{\Omega}\cdot\boldsymbol{\beta} + \boldsymbol{\Omega}\otimes\boldsymbol{\Omega} : \boldsymbol{\Theta}, \quad \alpha = \sin\pi x\sin\pi y, \quad \boldsymbol{\beta} = \begin{pmatrix}\alpha\\ \alpha\end{pmatrix}, \quad \boldsymbol{\Theta} = \begin{pmatrix}\alpha & \alpha\\ \alpha & \alpha\end{pmatrix}. \tag{5}$$



(a) Error as N increases in 1D

(b) Error as h decreases in 1D

(c) Error as N increases in 2D

(d) Error as h decreases in 2D

Figure 2: The MC Flux error as N increases or h decreases

Future Work

The boxed term in Eq. (3) may be causing the degradation in 2D because of noise, which we think we can dampen.

Result #2: Cost is independent of MFP

Figure 3 shows that our hybrid method (Nose*) doesn't slow as $\epsilon \to 0$ [11], but IMC slows as $1/\epsilon^2$.

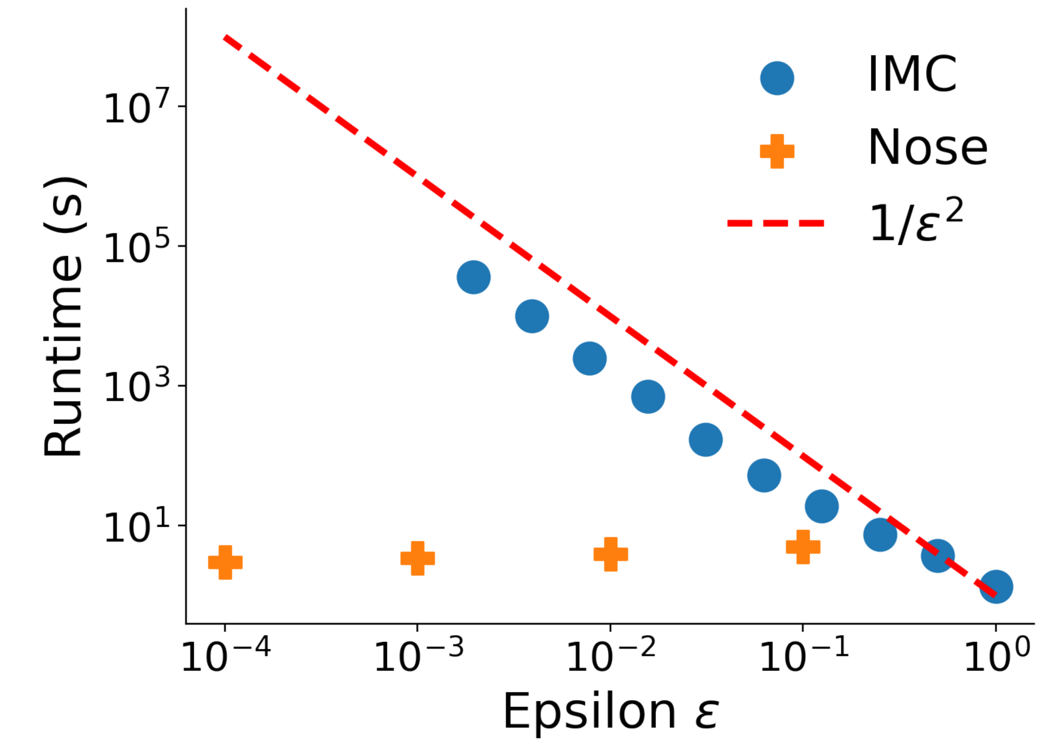


Figure 3: Runtime for thick diffusion limit calculation

*Nose is an acronym for **no s**cattering **e**vents.

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